ON THE NONNORMALITY OF SUBITERATION FOR FLUID-STRUCTURE INTERACTION PROBLEMS

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Fluid-structure interaction problems occur in many engineering sciences, including aerospace engineering, bio-mechanics and civil engineering. An essential property of these problems is that the position of the interface between the fluid and the structure is not known a priori. The interface position is interconnected with the state variables in the fluid and the structure by interface conditions. Moreover, the number of interface conditions is one more than the number of boundary conditions required by the separate boundary-value problems for the fluid and the structure. These characteristics render fluid-structure interaction a free-boundary problem.

The interconnection between the state variables and their domain of definition poses a fundamental problem in the numerical treatment of free-boundary problems. Consequently, Newton's method requires so-called *shape derivatives*, viz., the derivatives of the residuals of the boundary-value problem with respect to the free-boundary position. These shape derivatives render Newton's method prohibitively expensive for complex problems. Therefore, solution methods for free-boundary problems typically implement an iterative process based on partitioning: (1) the boundary-value problem(s) is (are) solved with a subset of the free-boundary conditions imposed, and (2) the free boundary is adjusted to relax the remaining free-boundary condition. Different names are used to designate this iterative procedure, e.g., subiteration, successive approximation or Picard iteration.

In the present work we consider subiteration for fluid-structure interaction problems. Our results are based on a model problem, viz., the piston problem. However, the main results extend to other fluid-structure interaction problems. A new fundamental result is that the subiteration operator is *nonnormal*. An important implication of this nonnormality is that the convergence behaviour of the method is generally nonmonotonous. In particular, the subiteration process can initially diverge before convergence occurs. The initial divergence can cause failure of the computational method despite formal stability. Moreover, the nonnormality impairs the effectivity of acceleration techniques such as Krylov subspace methods. Numerical experiments are presented to illustrate the theoretical results.